

# Solid Mechanics - 202041

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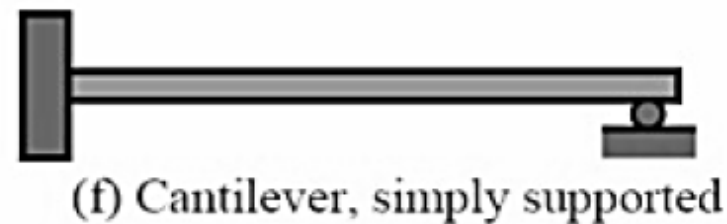
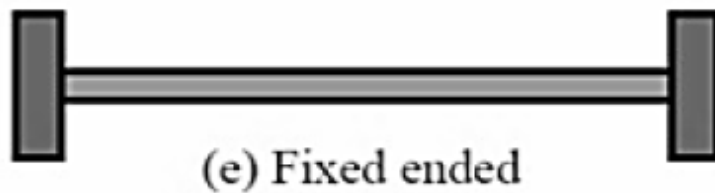
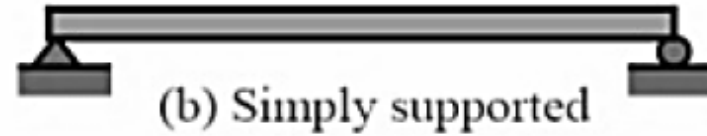
# Unit II Shear Force and Bending Moment Diagrams

CO2. DRAW Shear force and bending moment diagram for various types of transverse loading and support.

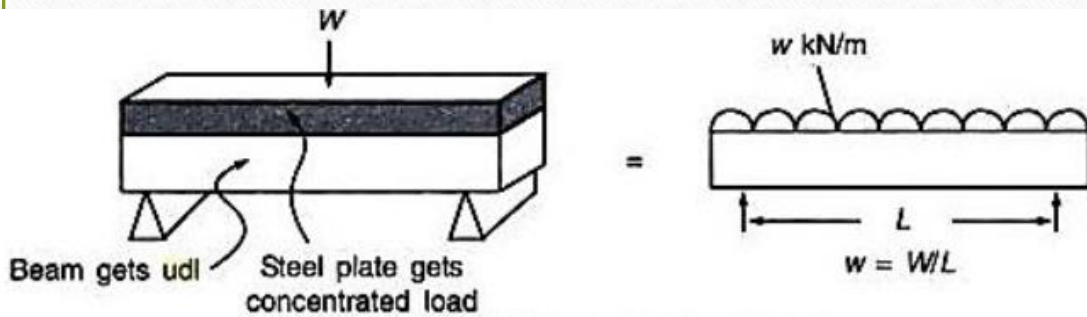
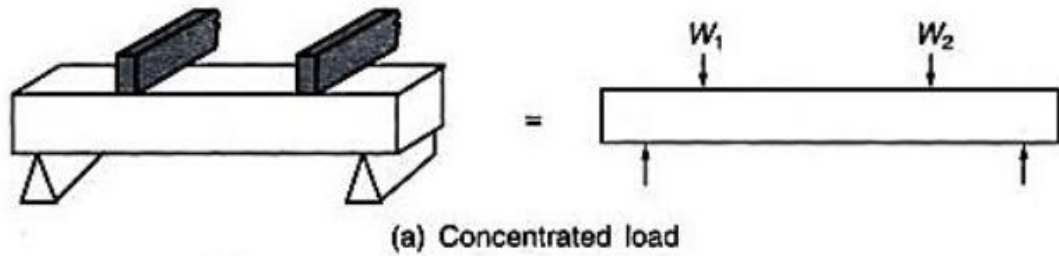
SFD & BMD: Introduction to SFD, BMD with application, SFD & BMD for statically determinate beam due to concentrated load, uniformly distributed load, uniformly varying load, couple and combined loading, Relationship between rate of loading, shear force and bending moment, Concept of zero shear force, Maximum bending moment, point of contra-flexure



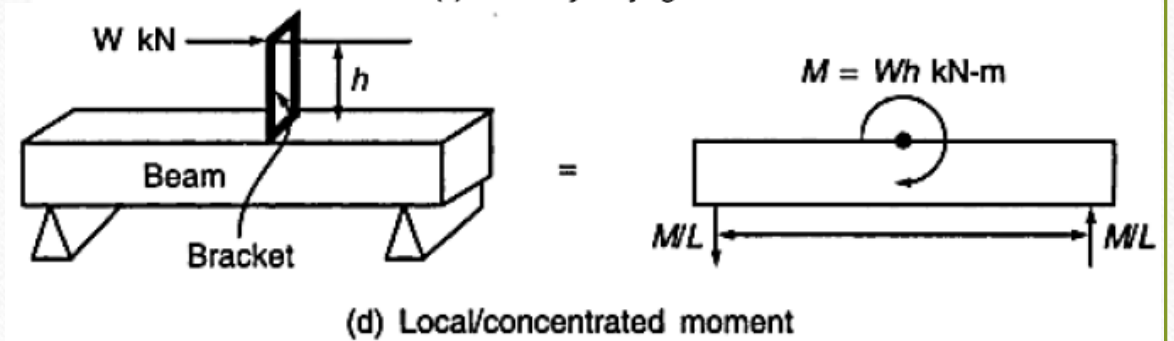
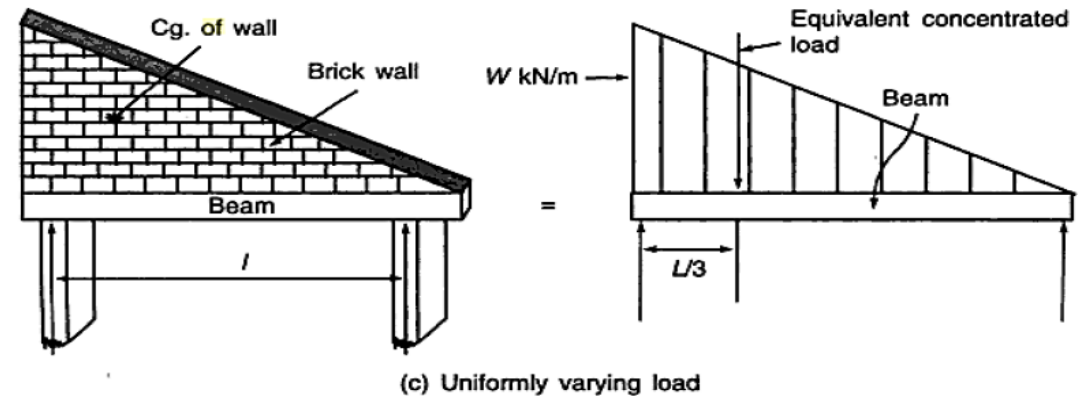
# Types of Beams



# Types of Loads



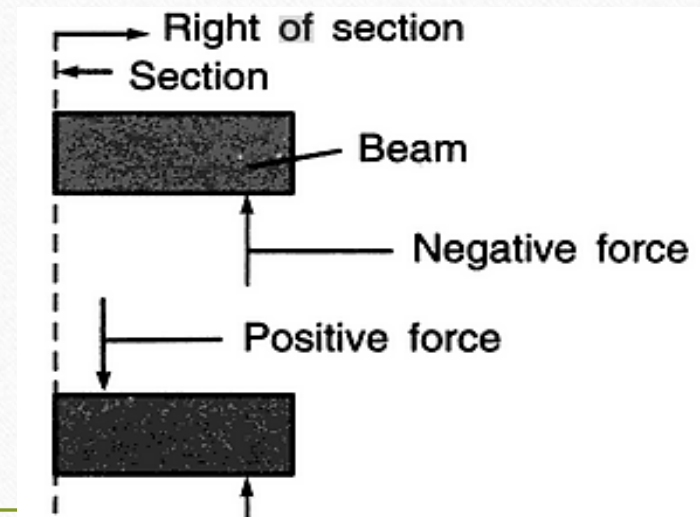
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# Shear Force Diagram (SFD)

- A *shear force (SF)* is defined as the algebraic sum of all the vertical forces, either to the left or to the right hand side of the section
- Shear Force Diagram: is graph connecting Shear Forces at various locations on the beam.

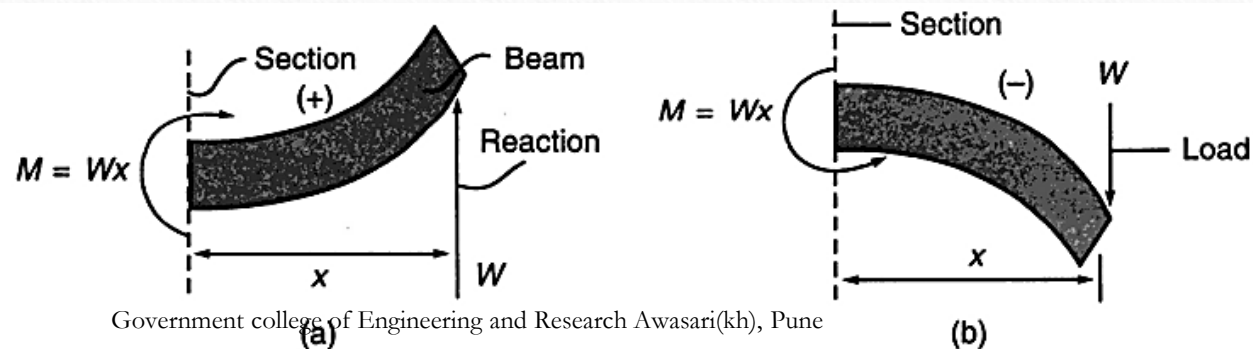
**A shear force which tends to rotate the beam in clockwise direction is positive and vice versa**



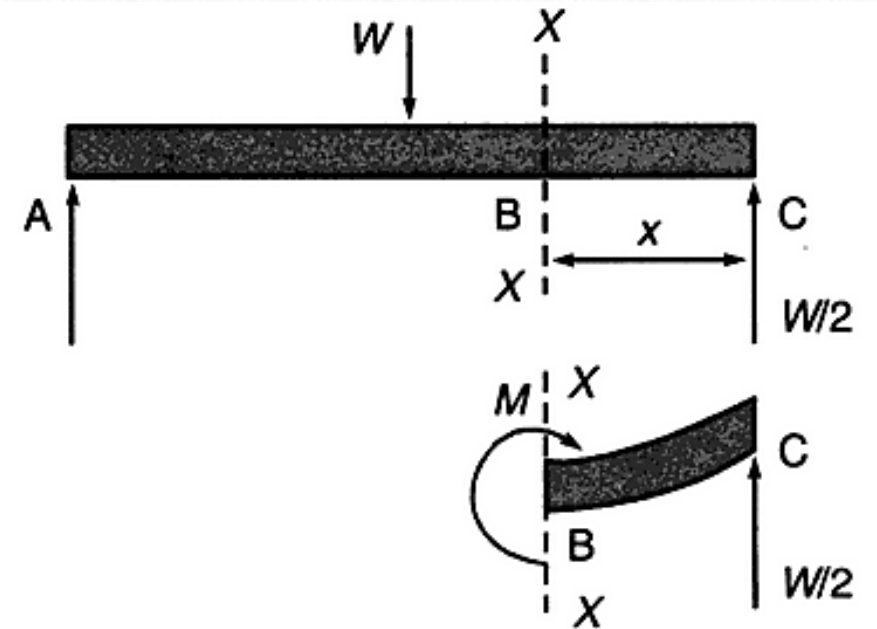


# Bending Moment Diagram (BMD)

- A *bending moment (BM)* is defined as the algebraic sum of the moments of all the forces either to the left or to the right of a section.
- BMD: Diagram is graph connecting bending moments at various locations



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# General Guidelines on Construction of SFD and BMD

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- The load, shear and bending moment diagrams should be constructed one below the other, in that order, all with the same horizontal scale.
- The dimension on the beam need not be scaled but should be relative and proportionate
- Ordinates (i.e., BM and SF values) need not be plotted to scale but should be relative. Curvature may need to be exaggerated for clarity.
- Principal ordinates (BM and SF values at salient points) should be labeled on both SFD and BMD.
- A clear distinction must be made on all straight lines as to whether the line is horizontal or has a positive or negative slope.



A simply supported beam AB of span 2.5 m is carrying two point loads as shown in Fig. 13.13.

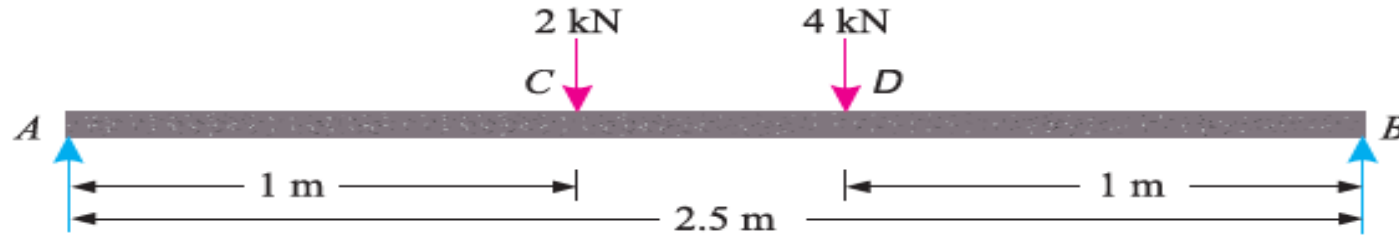


Fig. 13.13

Draw the shear force and bending moment diagrams for the beam.

**SOLUTION.** Given : Span ( $l$ ) = 2.5 m ; Point load at C ( $W_1$ ) = 2 kN and point load at B ( $W_2$ ) = 4 kN.

First of all let us find out the reactions  $R_A$  and  $R_B$ . Taking moments about A and equating the same,

$$R_B \times 2.5 = (2 \times 1) + (4 \times 1.5) = 8$$

$$R_B = 8/2.5 = 3.2 \text{ kN}$$

and

$$R_A = (2 + 4) - 3.2 = 2.8 \text{ kN}$$

**Shear force diagram**

The shear force diagram is shown in Fig. 13.14 (b) and the values are tabulated here:

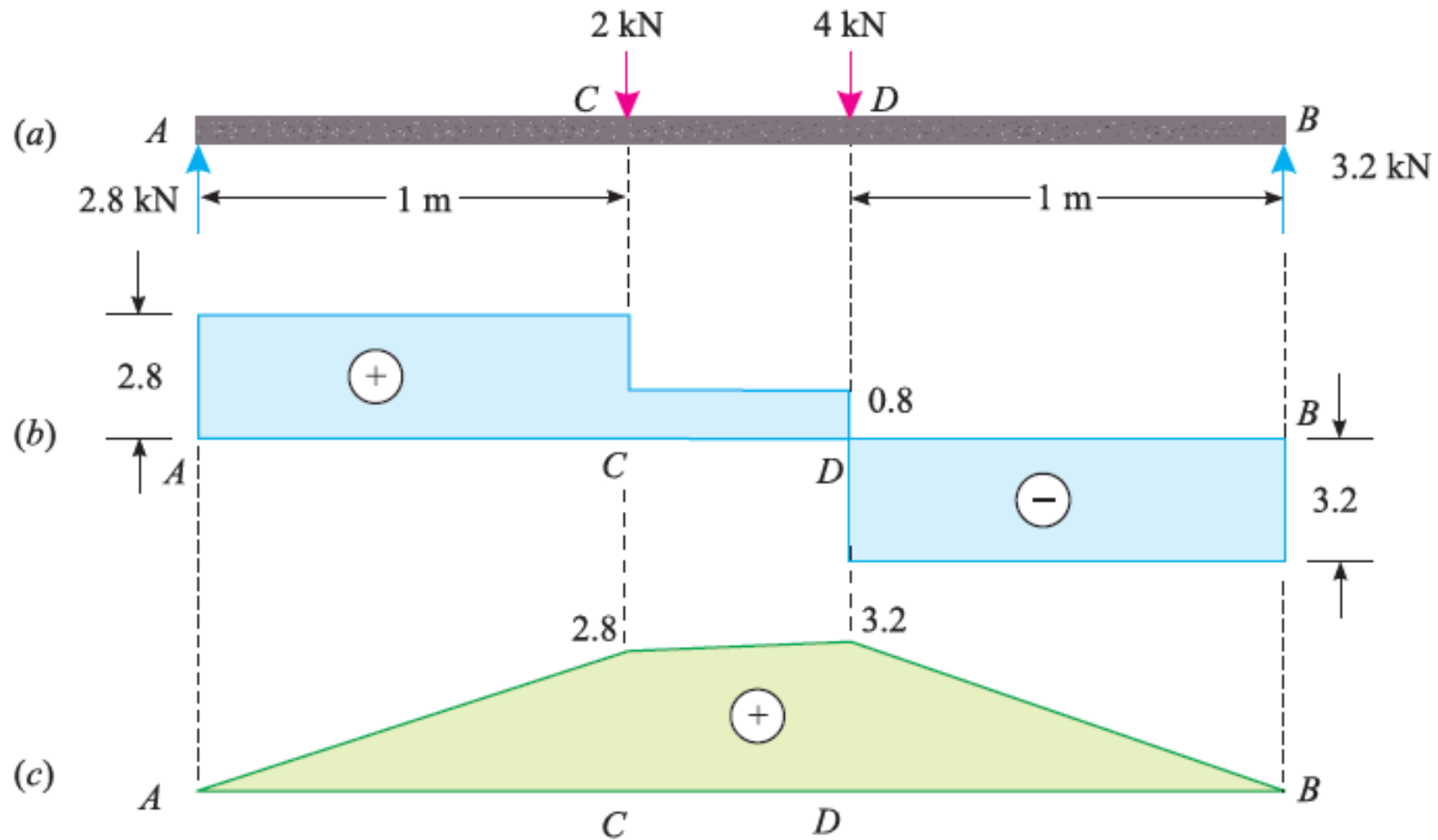
$$F_A = +R_A = 2.8 \text{ kN}$$

$$F_C = +2.8 - 2 = 0.8 \text{ kN}$$

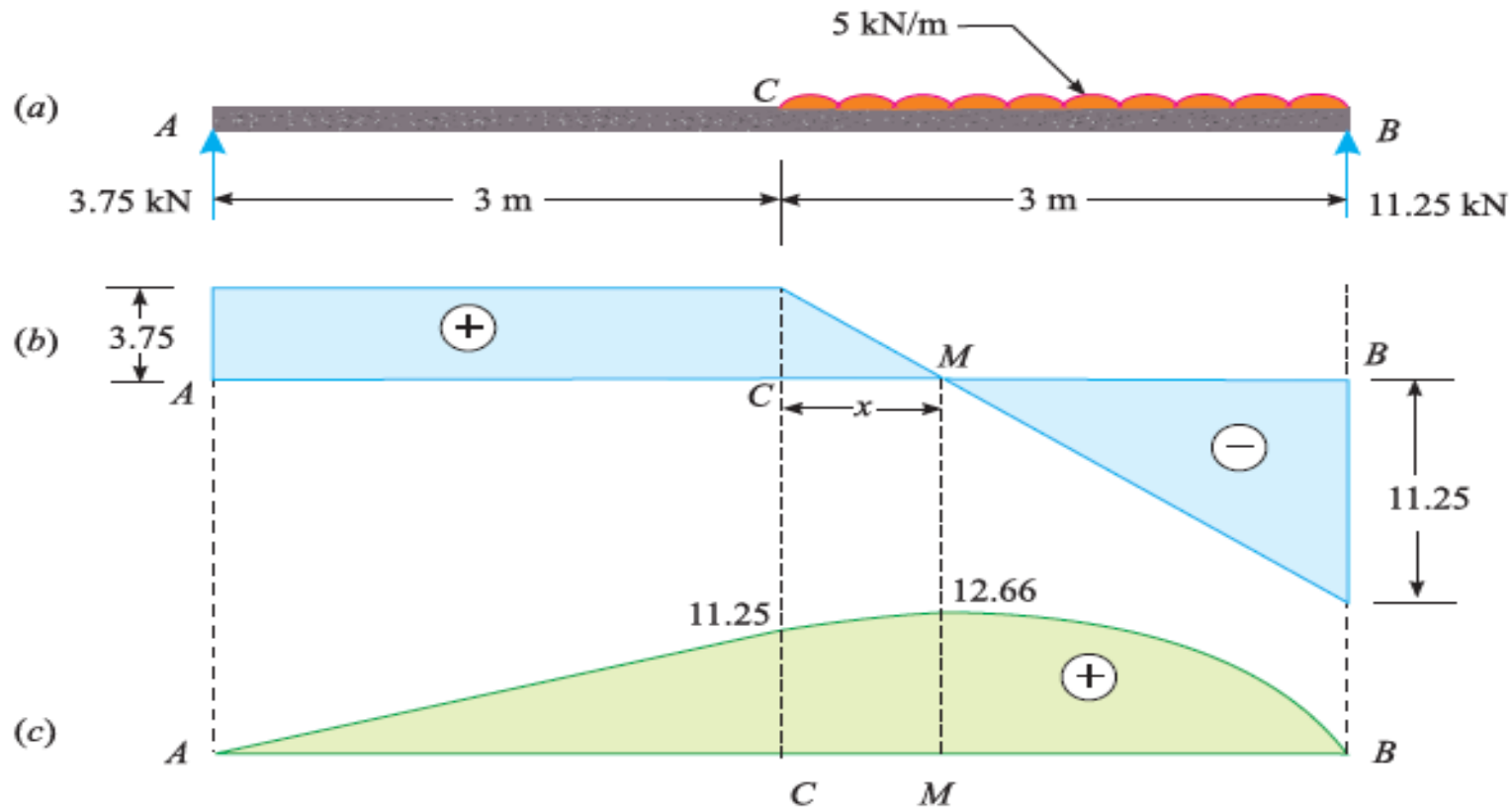
$$F_D = 0.8 - 4 = -3.2 \text{ kN}$$

$$F_B = -3.2 \text{ kN}$$





A simply supported beam 6 m long is carrying a uniformly distributed load of 5 kN/m over a length of 3 m from the right end. Draw the S.F. and B.M. diagrams for the beam and also calculate the maximum B.M. on the section.





**SOLUTION.** Given : Span ( $l$ ) = 6 m ; Uniformly distributed load ( $w$ ) = 5 kN/m and length of the beam  $CB$  carrying load ( $a$ ) = 3 m.

First of all, let us find out the reactions  $R_A$  and  $R_B$ . Taking moments about  $A$  and equating the same,

$$R_B \times 6 = (5 \times 3) \times 4.5 = 67.5$$

$$\therefore R_B = \frac{67.5}{6} = 11.25 \text{ kN}$$

and

$$R_A = (5 \times 3) - 11.25 = 3.75 \text{ kN}$$

### *Shear force diagram*

The shear force diagram is shown in Fig. 13.16 (b) and the values are tabulated here:

$$F_A = +R_A = +3.75 \text{ kN}$$

$$F_C = +3.75 \text{ kN}$$

$$F_B = +3.75 - (5 \times 3) = -11.25 \text{ kN}$$

A simply supported beam 5 m long is loaded with a uniformly distributed load of 10 kN/m over a length of 2 m as shown in Fig. 13.17.

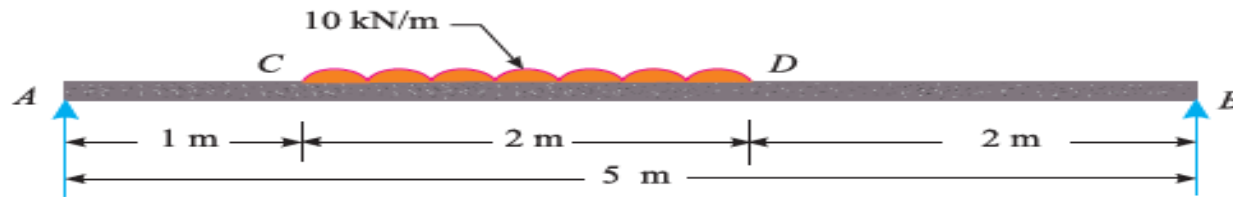


Fig. 13.17

Draw shear force and bending moment diagrams for the beam indicating the value of maximum bending moment.

**SOLUTION.** Given : Span ( $l$ ) = 5 m ; Uniformly distributed load ( $w$ ) = 10 kN/m and length of the beam  $CD$  carrying load ( $a$ ) = 2 m.

First of all, let us find out the reactions  $R_A$  and  $R_B$ . Taking moments about  $A$  and equating the same,

$$R_B \times 5 = (10 \times 2) \times 2 = 40$$

$$\therefore R_B = 40/5 = 8 \text{ kN}$$

and  $R_A = (10 \times 2) - 8 = 12 \text{ kN}$

**Shear force diagram**

The shear force diagram is shown in Fig. 13.18 (b) and the values are tabulated here:

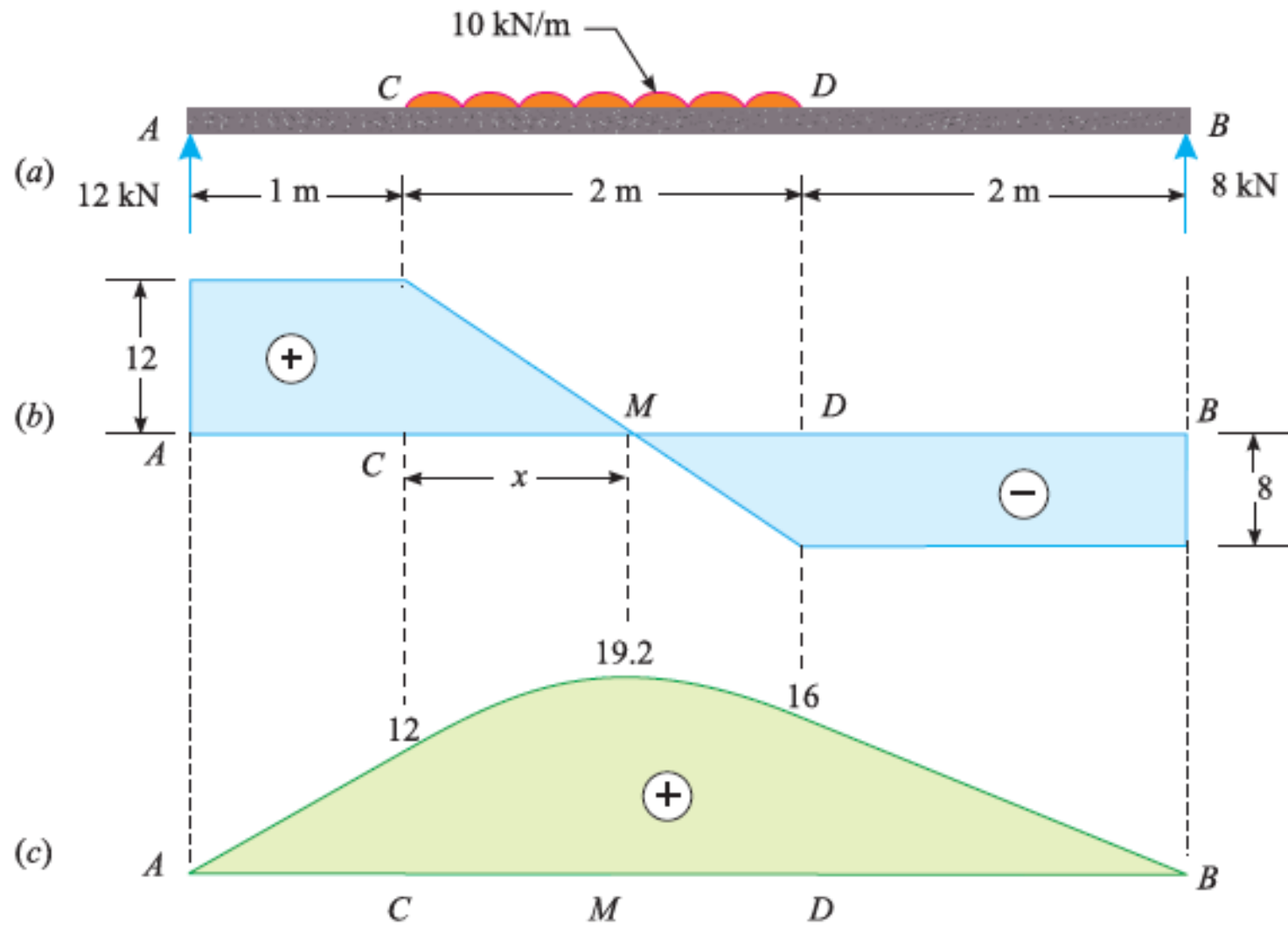
$$F_A = +R_A = +12 \text{ kN}$$

$$F_C = +12 \text{ kN}$$

$$F_D = +12 - (10 \times 2) = -8 \text{ kN}$$

$$F_B = -8 \text{ kN}$$





Draw shear force and bending moment diagrams for a cantilever beam of span 1.5 m carrying point loads as shown in Fig. 13.3 (a).

**SOLUTION.** Given : Span ( $l$ ) = 1.5 m ; Point load at B ( $W_1$ ) = 1.5 kN and point load at C ( $W_2$ ) = 2 kN.

### *Shear force diagram*

The shear force diagram is shown in Fig. 13.3 (b) and the values are tabulated here:

$$F_B = -W_1 = -1.5 \text{ kN}$$

$$F_C = -(1.5 + W_2) = -(1.5 + 2) = -3.5 \text{ kN}$$

$$F_A = -3.5 \text{ kN}$$

### *Bending moment diagram*

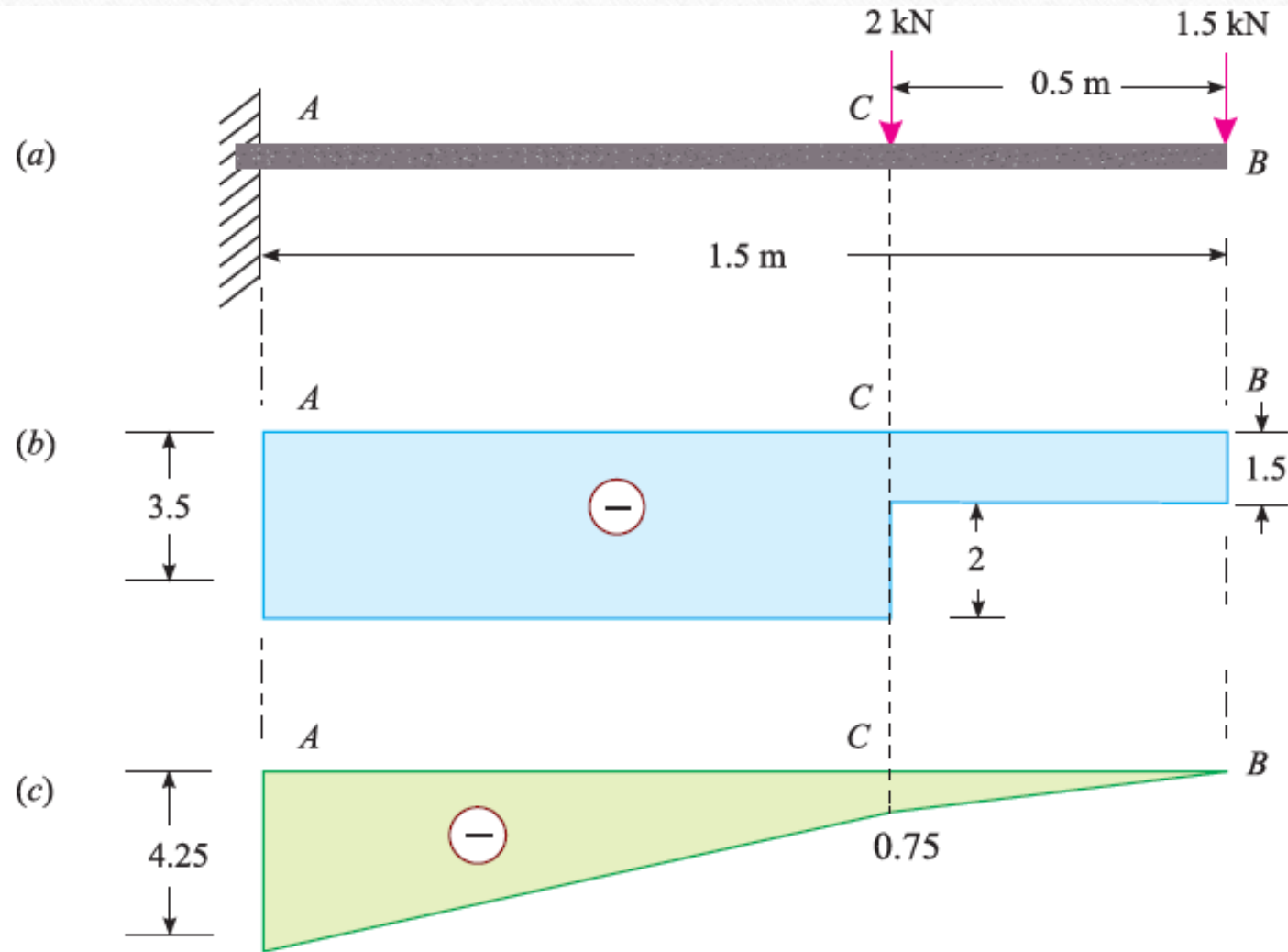
The bending moment diagram is shown in Fig. 13.3 (c) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -[1.5 \times 0.5] = -0.75 \text{ kN-m}$$

$$M_A = -[(1.5 \times 1.5) + (2 \times 1)] = -4.25 \text{ kN-m}$$





A cantilever beam AB, 2 m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw shear force and bending moment diagrams for the beam.

**SOLUTION.** Given : span ( $l$ ) = 2 m ; Uniformly distributed load ( $w$ ) = 1.5 kN/m and length of the cantilever CB carrying load ( $a$ ) = 1.6 m.

### *Shear force diagram*

The shear force diagram is shown in Fig. 13.5 (b) and the values are tabulated here:

$$F_B = 0$$

$$F_C = -w \cdot a = -1.5 \times 1.6 = -2.4 \text{ kN}$$

$$F_A = -2.4 \text{ kN}$$

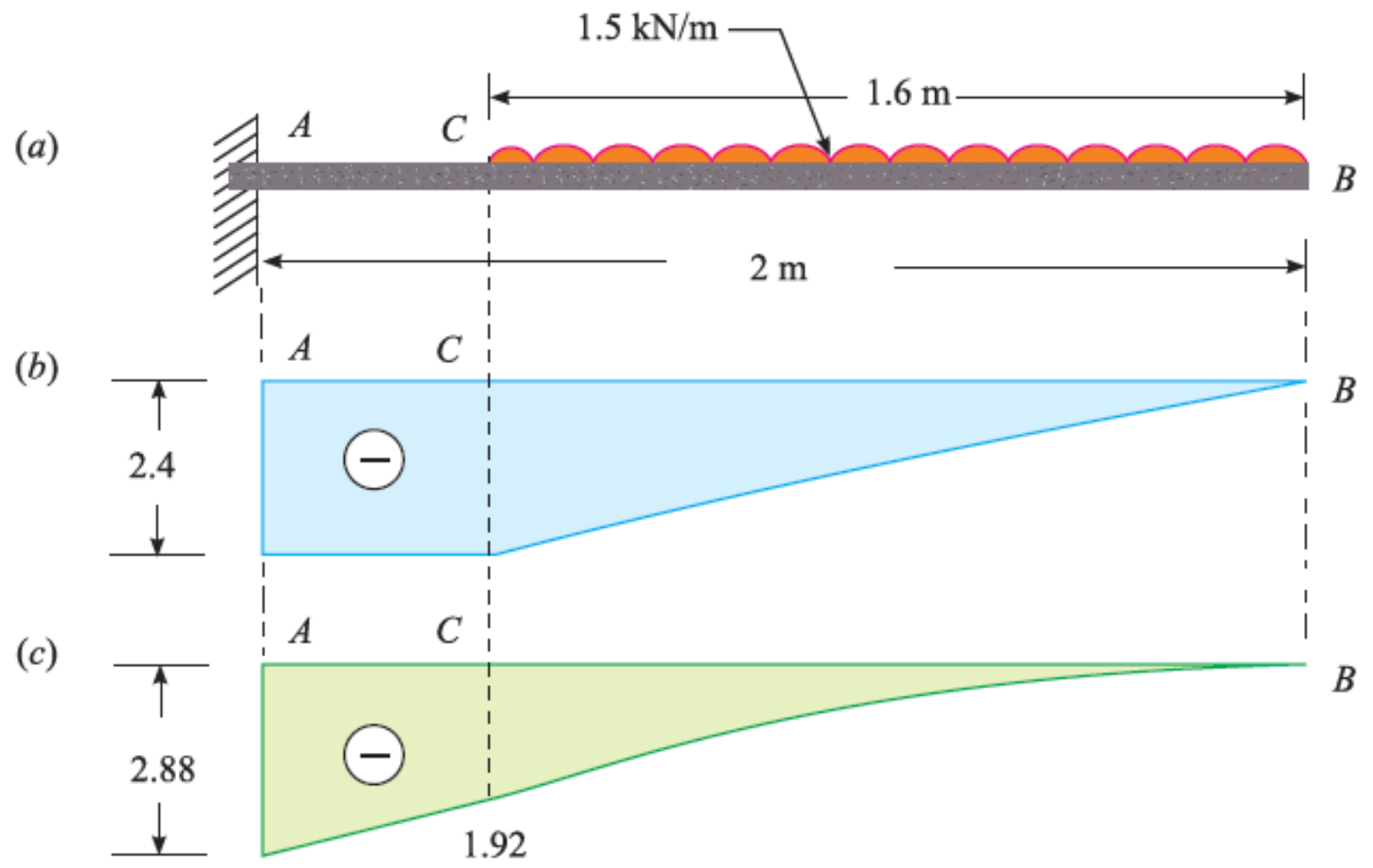
### *Bending moment diagram*

The bending moment diagram is shown in Fig. 13.5 (c) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -\frac{wa^2}{2} = \frac{1.5 \times (1.6)^2}{2} = -1.92 \text{ kN-m}$$

$$M_A = -\left[ (1.5 \times 1.6) \left( 0.4 + \frac{1.6}{2} \right) \right] = -2.88 \text{ kN-m}$$





A cantilever beam of 1.5 m span is loaded as shown in Fig. 13.6 (a). Draw the shear force and bending moment diagrams.

**SOLUTION.** Given : Span ( $l$ ) = 1.5 m ; Point load at B ( $W$ ) = 2 kN ; Uniformly distributed load ( $w$ ) = 1 kN/m and length of the cantilever AC carrying the load ( $a$ ) = 1 m.

The shear force diagram is shown in Fig. 13.6 (b) and the values are tabulated here:

$$F_B = -W = -2 \text{ kN}$$

$$F_C = -2 \text{ kN}$$

$$F_A = -[2 + (1 \times 1)] = -3 \text{ kN}$$

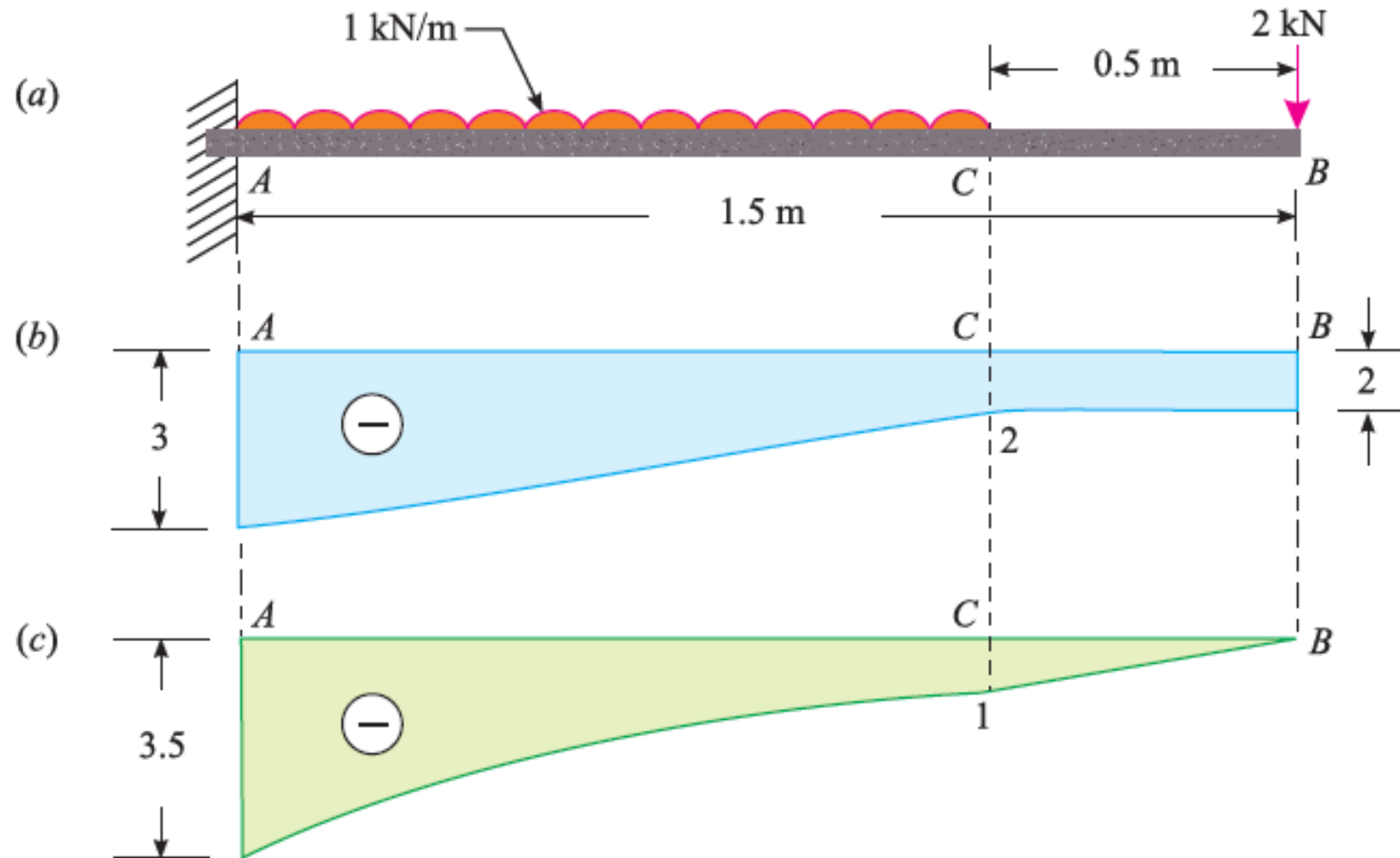
### *Bending moment diagram*

The bending moment diagram is shown in Fig. 13.6 (c) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -[2 \times 0.5] = -1 \text{ kN-m}$$

$$M_A = -\left[(2 \times 1.5) + (1 \times 1) \times \frac{1}{2}\right] = -3.5 \text{ kN-m}$$



A cantilever beam 4 m long carries a gradually varying load, zero at the free end to 3 kN/m at the fixed end. Draw B.M. and S.F. diagrams for the beam.

**SOLUTION.** Given : Span ( $l$ ) = 4 m and gradually varying load at A ( $w$ ) = 3 kN/m

The cantilever beam is shown in Fig. 13.8 (a).

### *Shear force diagram*

The shear force diagram is shown in Fig. 13.8 (b) and the values are tabulated here:

$$F_B = 0$$

$$F_A = -\frac{3 \times 4}{2} = -6 \text{ kN}$$

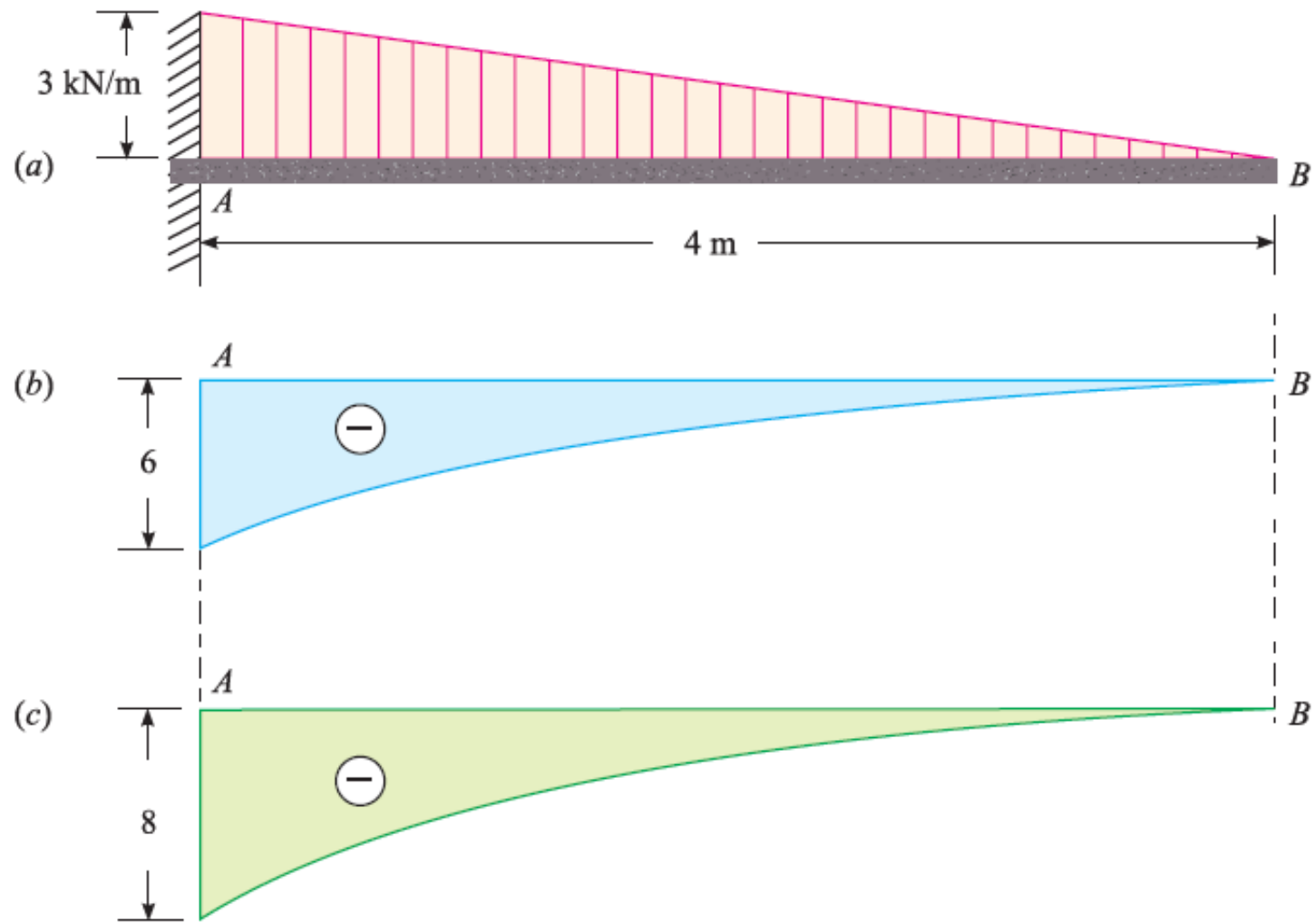
### *Bending moment diagram*

The bending moment diagram is shown in Fig. 13.8 (c) and the values are tabulated here:

$$M_B = 0$$

$$M_A = -\frac{3 \times (4)^2}{6} = -8 \text{ kN-m}$$





An overhanging beam ABC is loaded as shown in Fig. 13.30.

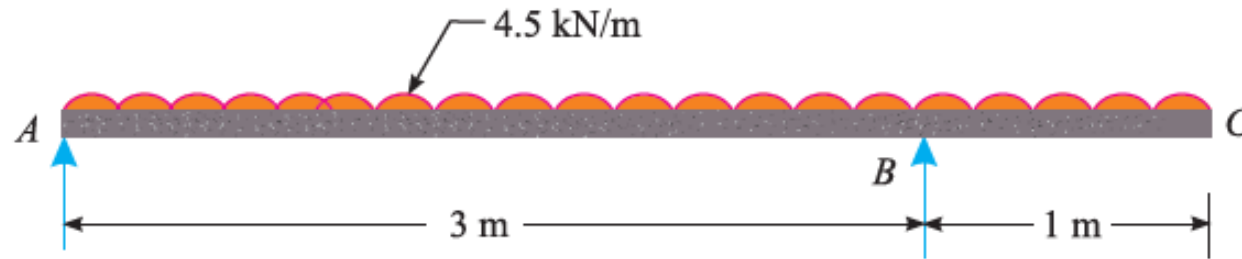


Fig. 13.30

Draw the shear force and bending moment diagrams and find the point of contraflexure, if any.

**SOLUTION.** Given : Span ( $l$ ) = 4 m ; Uniformly distributed load ( $w$ ) = 4.5 kN/m and overhanging length ( $c$ ) = 1 m.

First of all, let us find out the reactions  $R_A$  and  $R_B$ . Taking moment about A and equating the same,

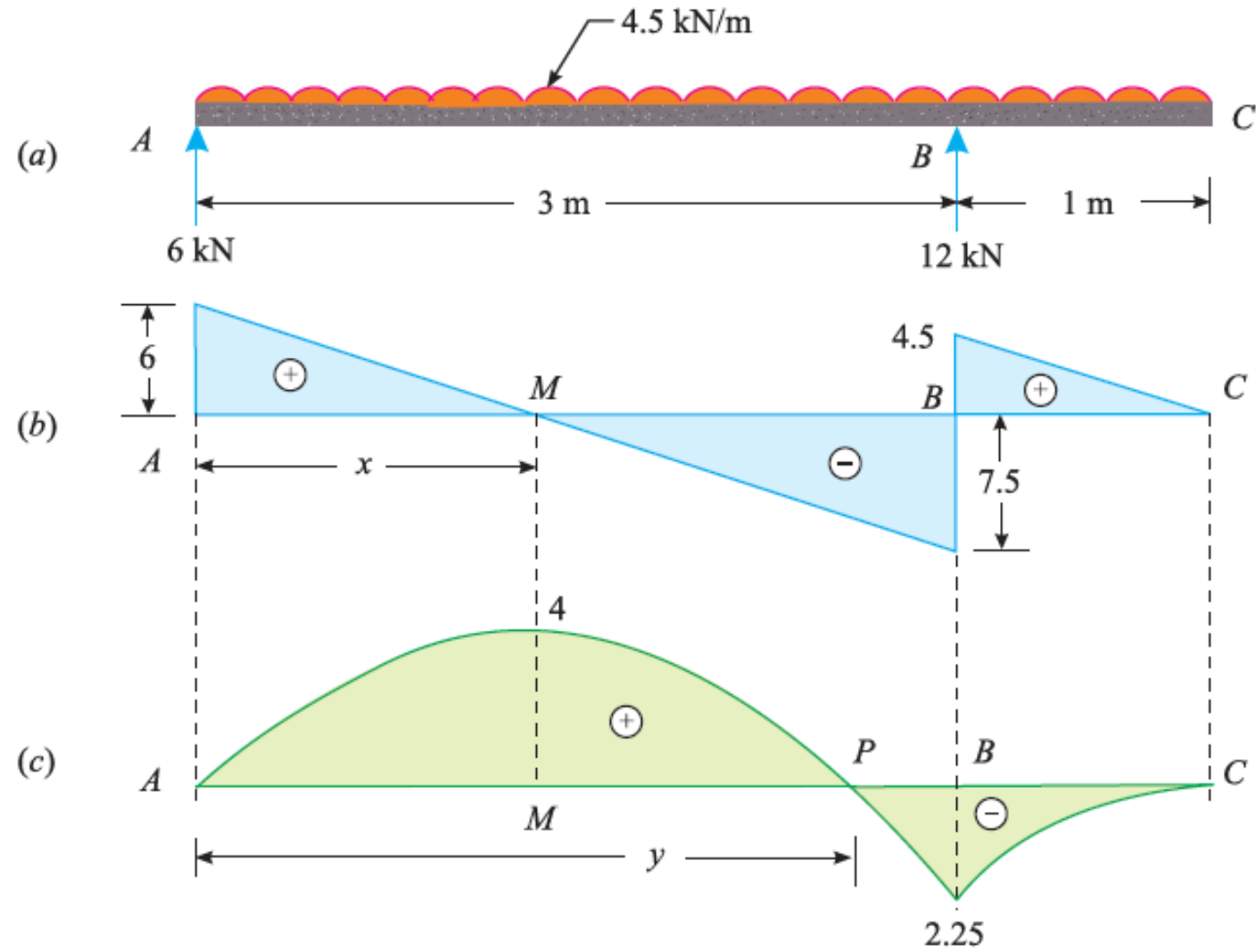
$$R_B \times 3 = (4.5 \times 4) \times 2 = 36$$

$$\therefore R_B = 36/3 = 12 \text{ kN}$$

$$\text{and } R_A = (4.5 \times 4) - 12 = 6 \text{ kN}$$

**Shear force diagram**

The shear force diagram is shown in Fig. 13.31 (b) and the values are tabulated here:





$$F_A = +R_A = +6 \text{ kN}$$

$$F_B = +6 - (4.5 \times 3) + 12 = 4.5 \text{ kN}$$

$$F_C = +4.5 - (4.5 \times 1) = 0$$

### *Bending moment diagram*

The bending moment diagram is shown in Fig. 13.31 (c) and the values are tabulated here:

$$M_A = 0$$

$$M_B = -\left(4.5 \times 1 \times \frac{1}{2}\right) = -2.25 \text{ kN-m}$$

$$M_C = 0$$

We know that maximum bending moment will occur at  $M$ , where the shear force changes sign. Let  $x$  be the distance between  $A$  and  $M$ . From the geometry of the figure between  $A$  and  $B$ , we find that

A beam ABCD, 4 m long is overhanging by 1 m and carries load as shown in Fig. 13.32.

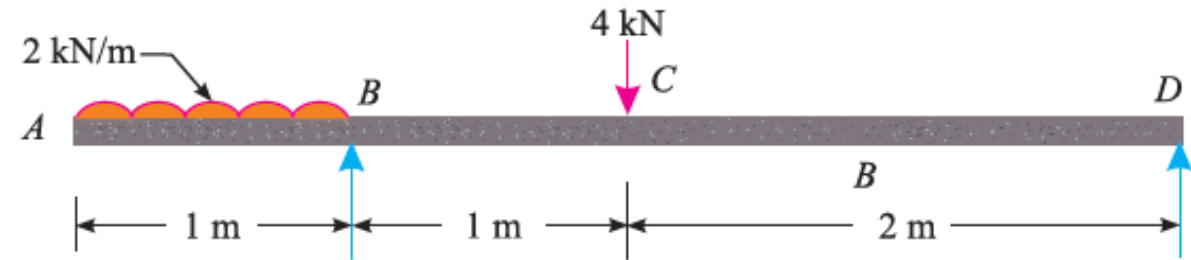


Fig. 13.32

Draw the shear force and bending moment diagrams for the beam and locate the point of contraflexure.

**SOLUTION.** Given : Span ( $l$ ) = 4 m ; Uniformly distributed load over AB ( $w$ ) = 2 kN/m and point load at C ( $W$ ) = 4 kN.

First of all, let us find out the reactions  $R_B$  and  $R_D$ . Taking moments about B and equating the same,

$$R_D \times 3 = (4 \times 1) - (2 \times 1) \times \frac{1}{2} = 3$$

$\therefore$

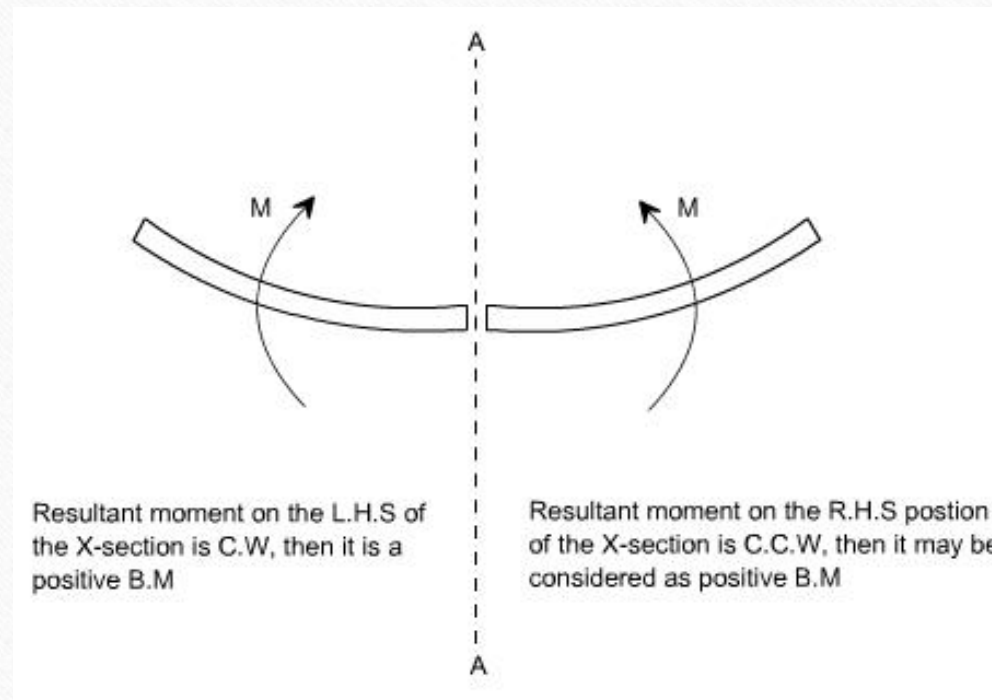
$$R_D = 3/3 = 1 \text{ kN}$$

and

$$R_B = (2 \times 1) + 4 - 1 = 5 \text{ kN}$$

# Sign Conventions for the Bending Moment:

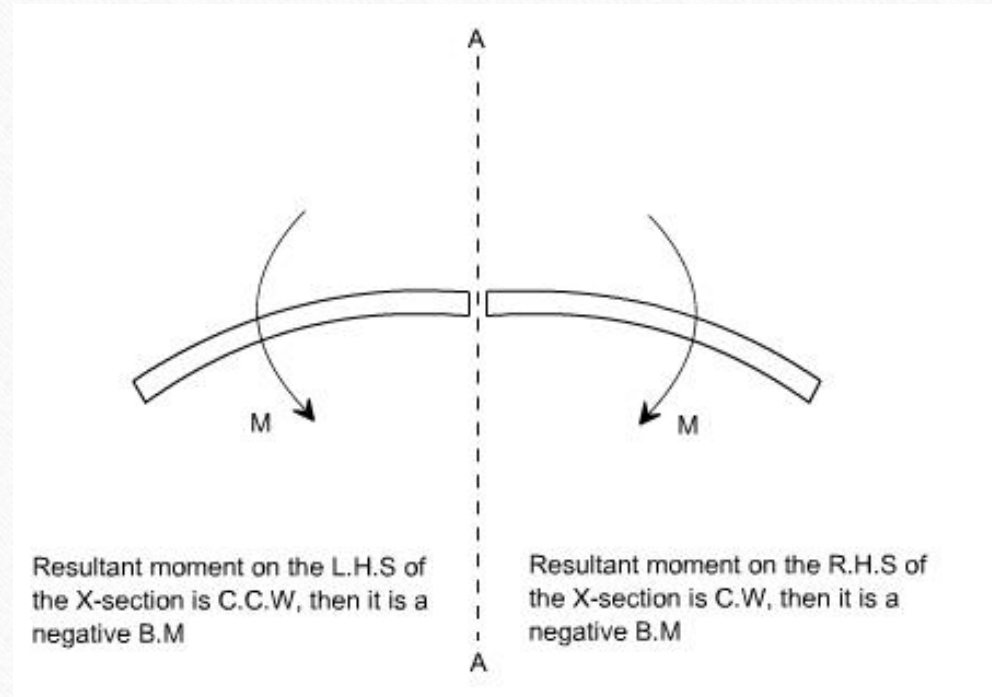
## a) Positive Bending Moment (Sagging)





## b) Negative Bending Moment(Hogging)

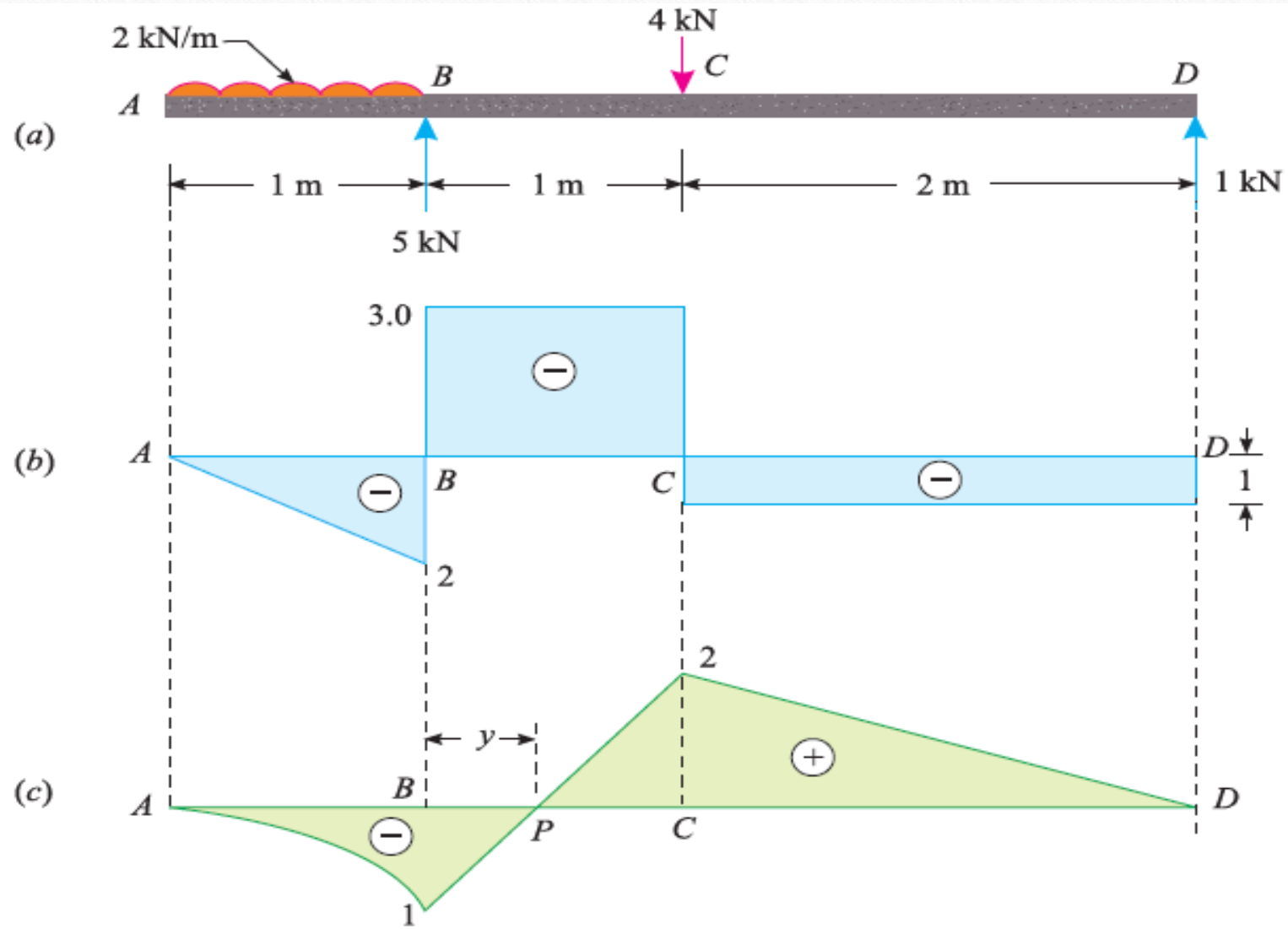
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# Point of Contraflexure:

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- It corresponds to a point where the bending moment changes the sign, hence in order to find the point of contraflexures obviously the B.M would change its sign when it cuts the X-axis therefore to get the points of contraflexure equate the bending moment equation equal to zero. The fibre stress is zero at such sections





### *Shear force diagram*

The shear force diagram is shown in Fig. 13.33 (b) and the values are tabulated here:

$$F_A = 0$$

$$F_B = 0 - (2 \times 1) + 5 = +3 \text{ kN}$$

$$F_C = +3 - 4 = -1 \text{ kN}$$

$$F_D = 1 \text{ kN}$$

### *Bending moment diagram*

The bending moment diagram is shown in Fig. 13.33 (c) and the values are tabulated here:

$$M_A = 0$$

$$M_B = -(2 \times 1) 0.5 = -1 \text{ kN-m}$$

$$M_C = 1 \times 2 = +2 \text{ kN}$$

$$M_D = 0$$

We know that maximum bending moment occurs either at  $B$  or  $C$ , where the shear force changes sign. From the geometry of the bending moment diagram, we find that maximum negative bending moment occurs at  $B$  and maximum positive bending moment occurs at  $C$ .

### *Point of contraflexure*

Let  $P$  be the point of contraflexure at a distance  $y$  from the support  $B$ . From the geometry of the figure between  $B$  and  $C$ , we find that

$$\frac{y}{1.0} = \frac{1-y}{2.0}$$

$$2y = 1 - y \quad \text{or} \quad 3y = 1$$

or

$$y = 1/3 = 0.33 \text{ m} \quad \text{Ans.}$$

Draw shear force and bending moment diagrams for the beam shown in Fig. 13.34. Indicate the numerical values at all important sections.

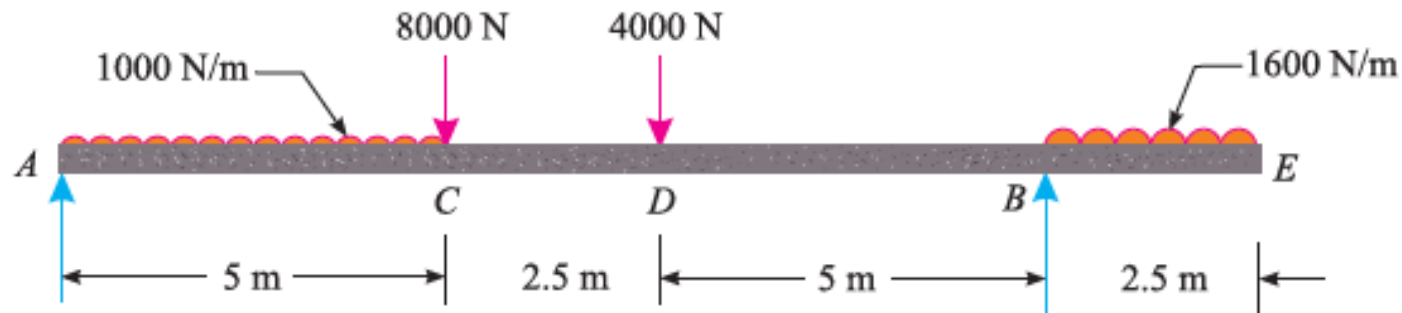


Fig. 13.34

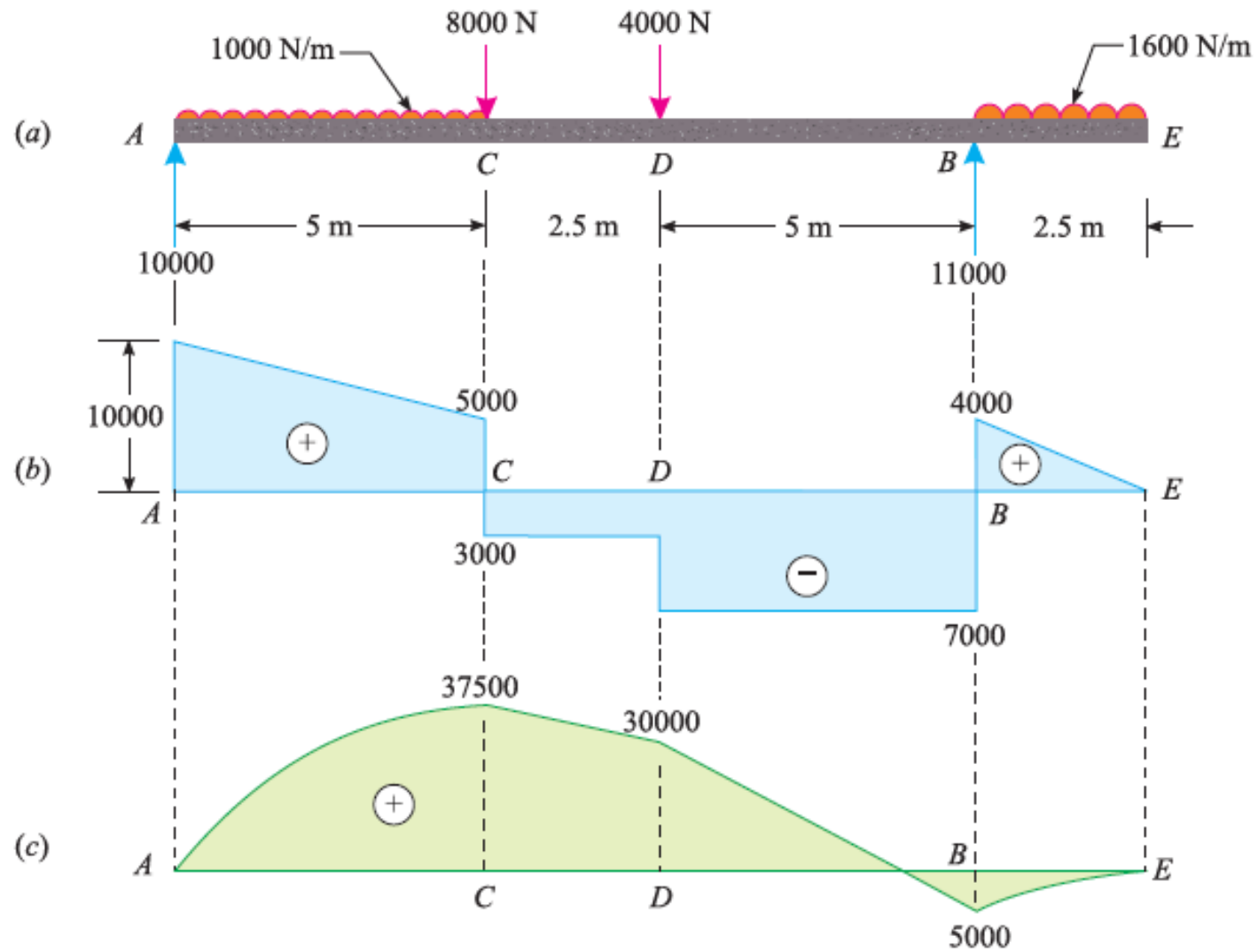
**SOLUTION.** Given : Span ( $l$ ) = 15 m ; Uniformly distributed load between A and B ( $w_1$ ) = 1000 N/m ; Point load at C ( $W_1$ ) = 8000 N ; Point load at D ( $W_2$ ) = 4000 N and uniformly distributed load between B and E ( $w_2$ ) = 1600 N/m.

First of all, let us find out the reactions  $R_A$  and  $R_B$ .

Taking moments about A and equating the same,

$$\begin{aligned} R_B \times 12.5 &= (1600 \times 2.5) \times 13.75 + (4000 \times 7.5) + (8000 \times 5) + (1000 \times 5) \times 2.5 \\ &= 137500 \end{aligned}$$

$$R_B = \frac{137500}{12.5} = 110000 \text{ N}$$





and

$$\begin{aligned}R_A &= (1000 \times 5 + 8000 + 4000 + 1600 \times 2.5) - 11000 \text{ N} \\ &= 10000 \text{ N}\end{aligned}$$

### *Shear force*

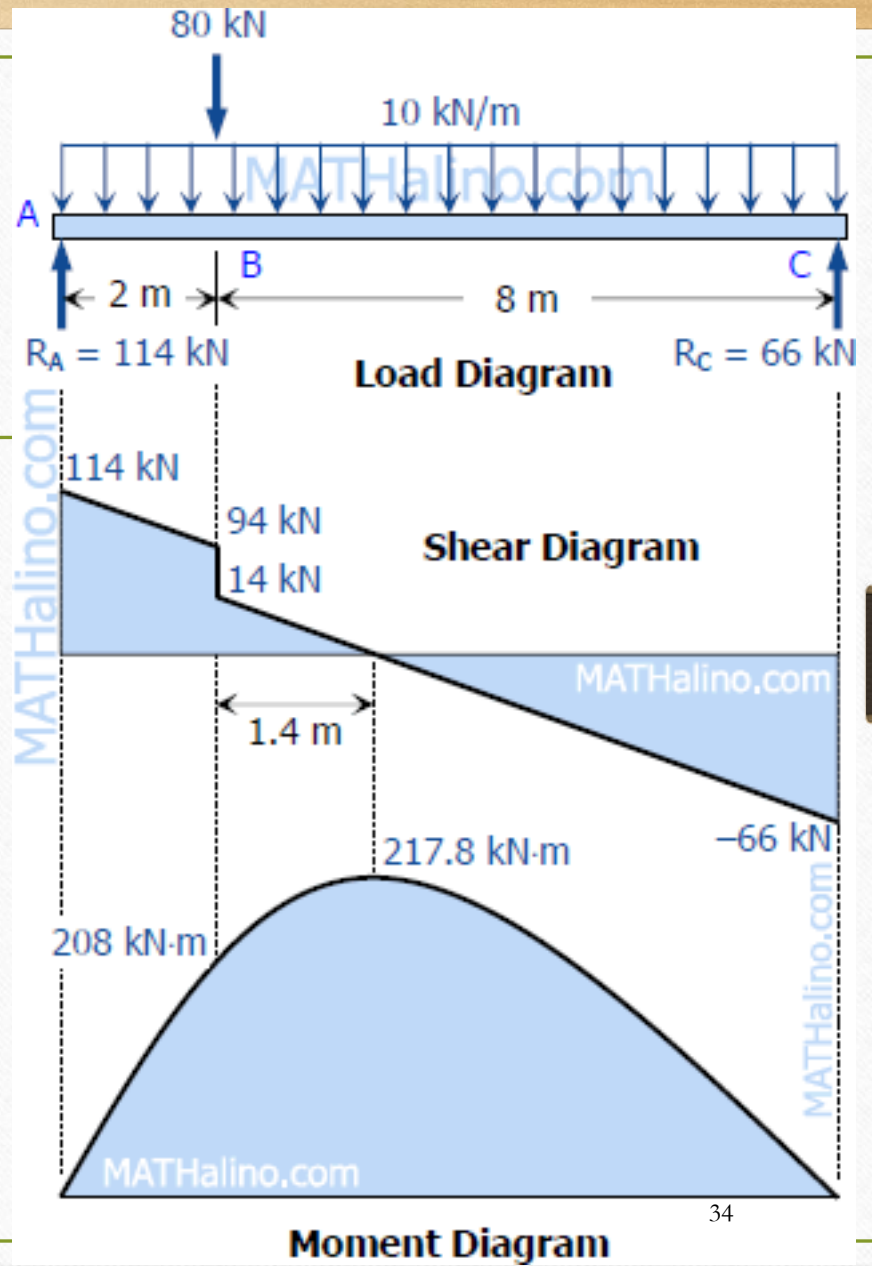
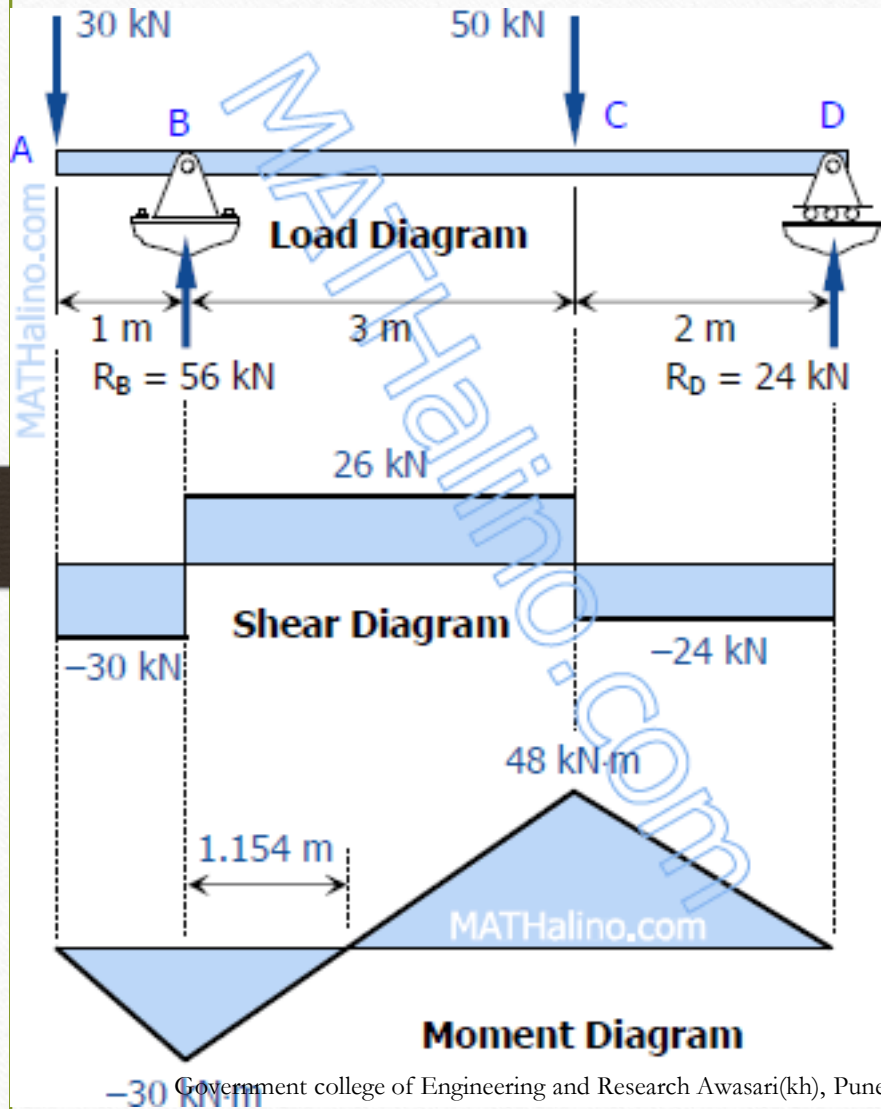
The shear force diagram is shown in Fig. 13.35 (b) and the values are tabulated here:

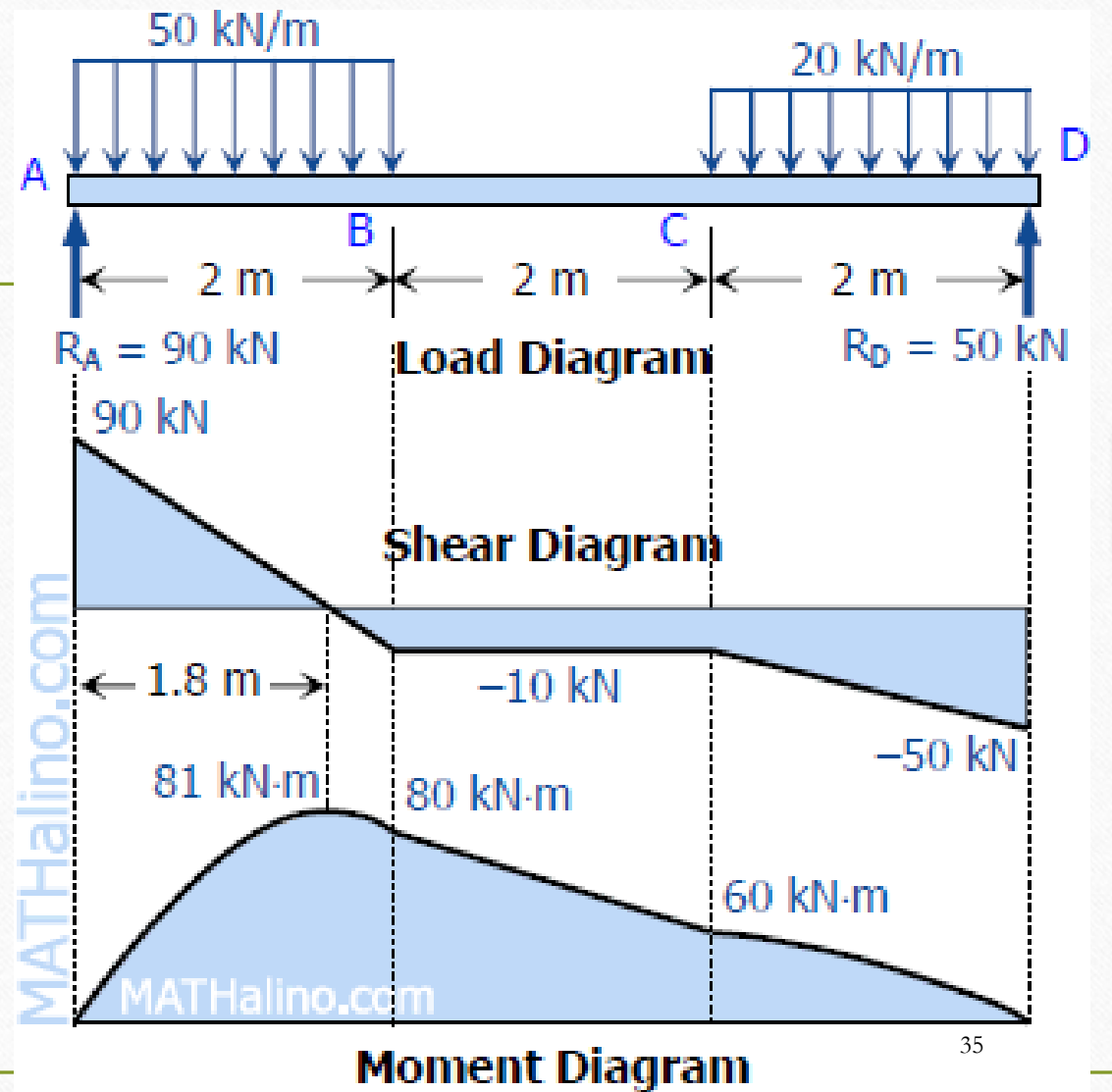
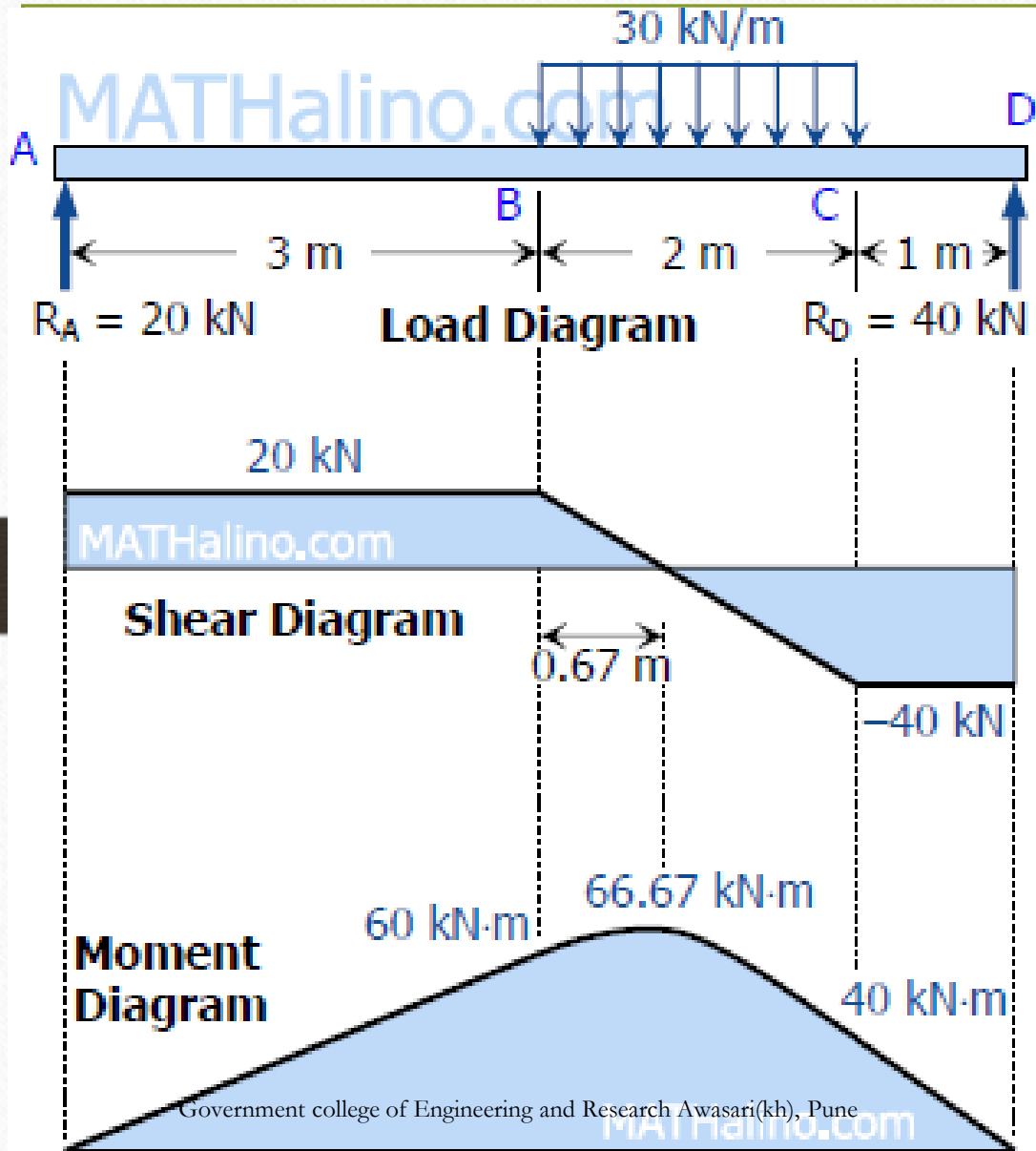
$$\begin{aligned}F_A &= + 10000 \text{ N} \\ F_C &= + 10000 - (1000 \times 5) - 800 = - 3000 \text{ N} \\ F_D &= - 3000 - 4000 = - 7000 \text{ N} \\ F_B &= - 7000 + 11000 = + 4000 \text{ N} \\ F_E &= + 4000 - 1600 \times 2.5 = 0\end{aligned}$$

### *Bending moment*

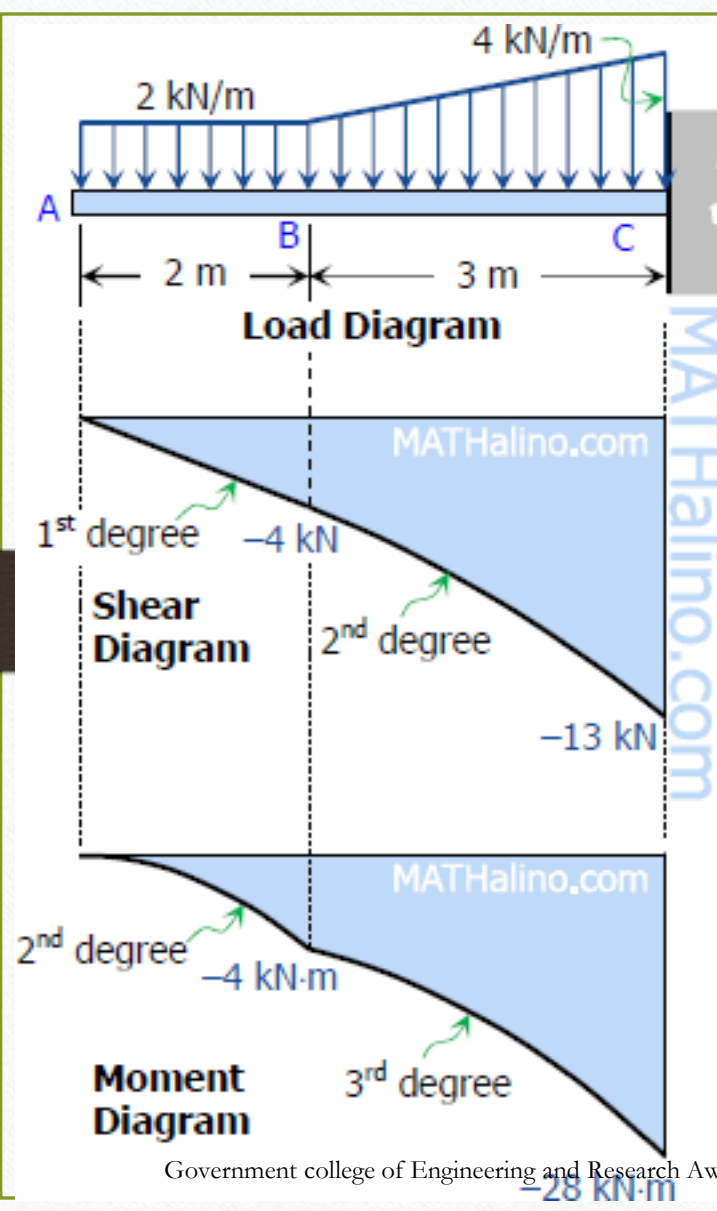
The bending moment diagram is shown in Fig. 13.35 (c), and the values are tabulated here:

$$\begin{aligned}M_A &= 0 \\ M_C &= (10000 \times 5) - (1000) \times (5 \times 2.5) = 37500 \text{ N-m} \\ M_D &= (10000 \times 7.5) - (1000 \times 5 \times 5) - (8000 \times 2.5) \text{ N-m} \\ &= 30000 \text{ N-m} \\ M_B &= - 1600 \times 2.5 \times \frac{2.5}{2} = - 5000 \text{ N-m}\end{aligned}$$

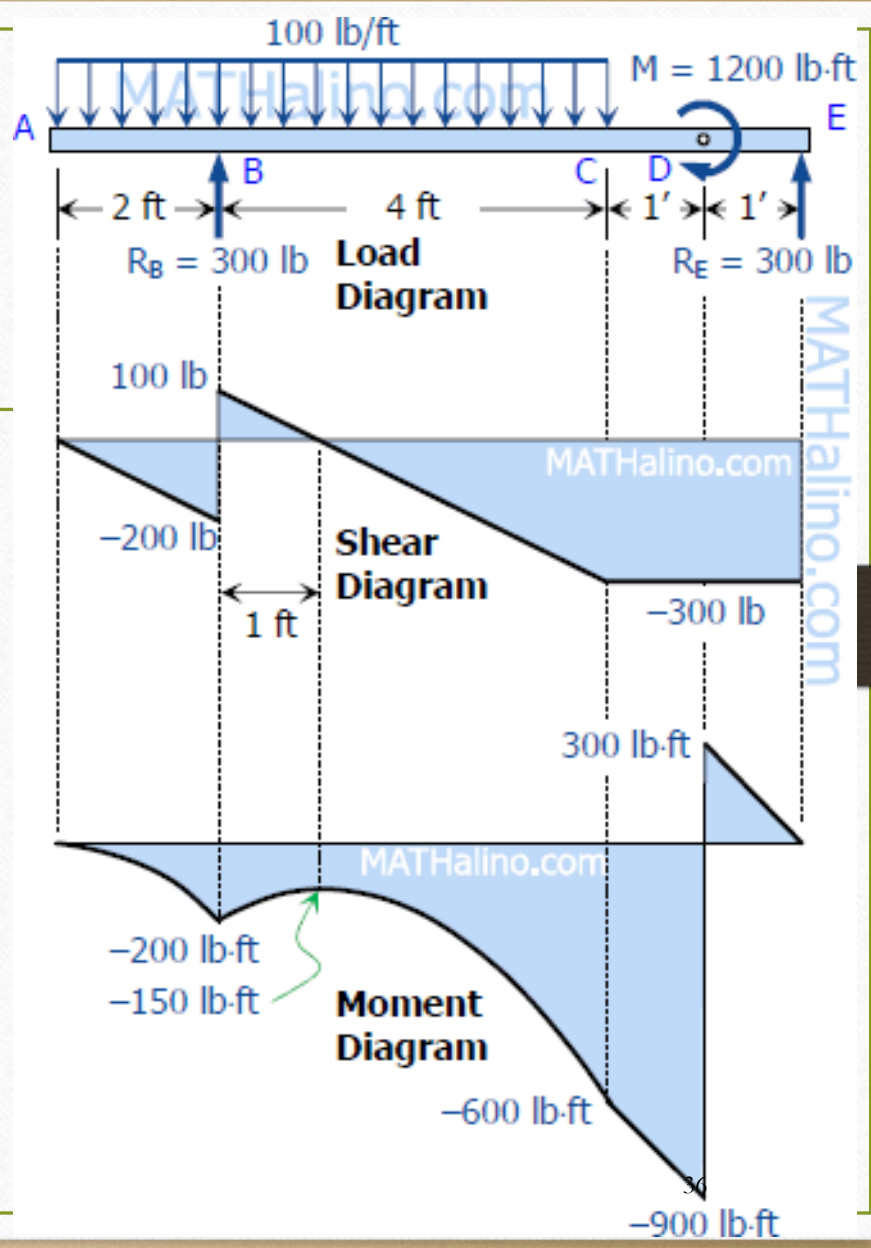


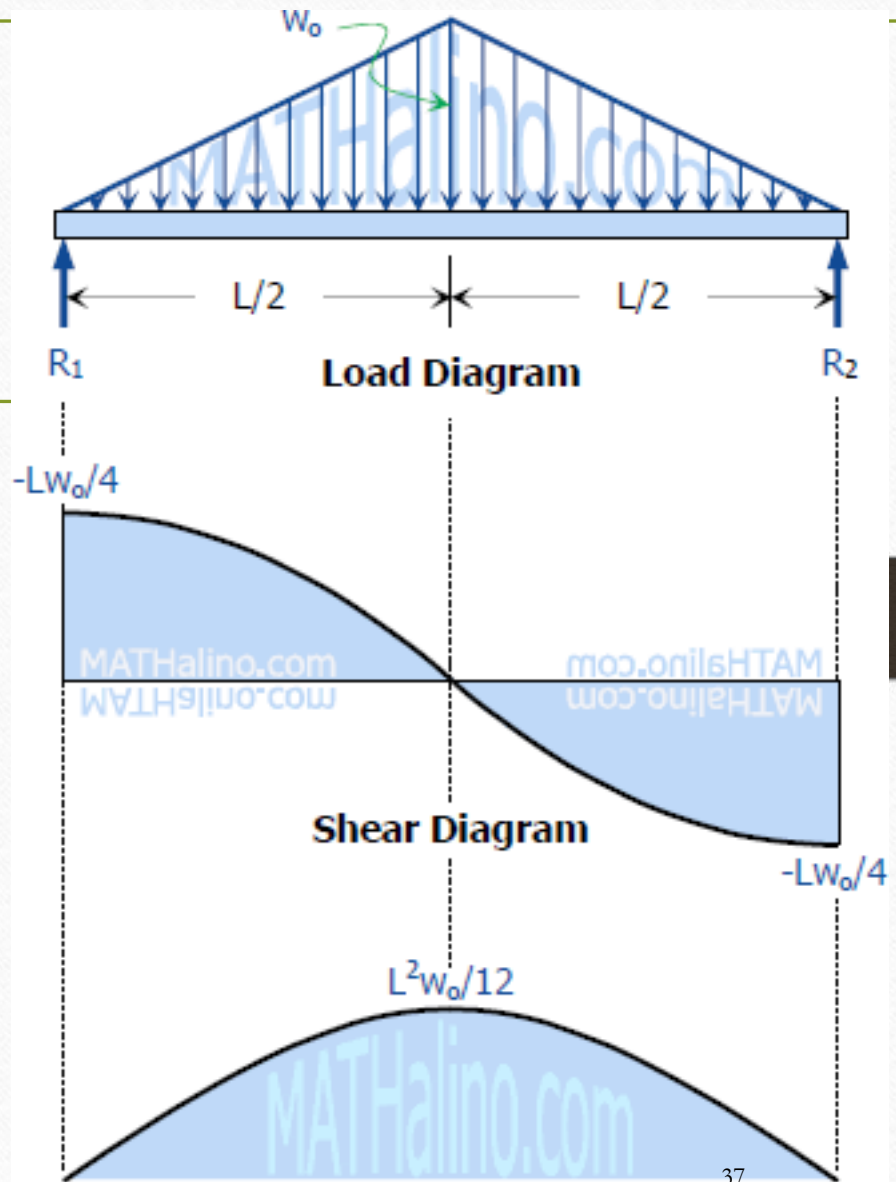
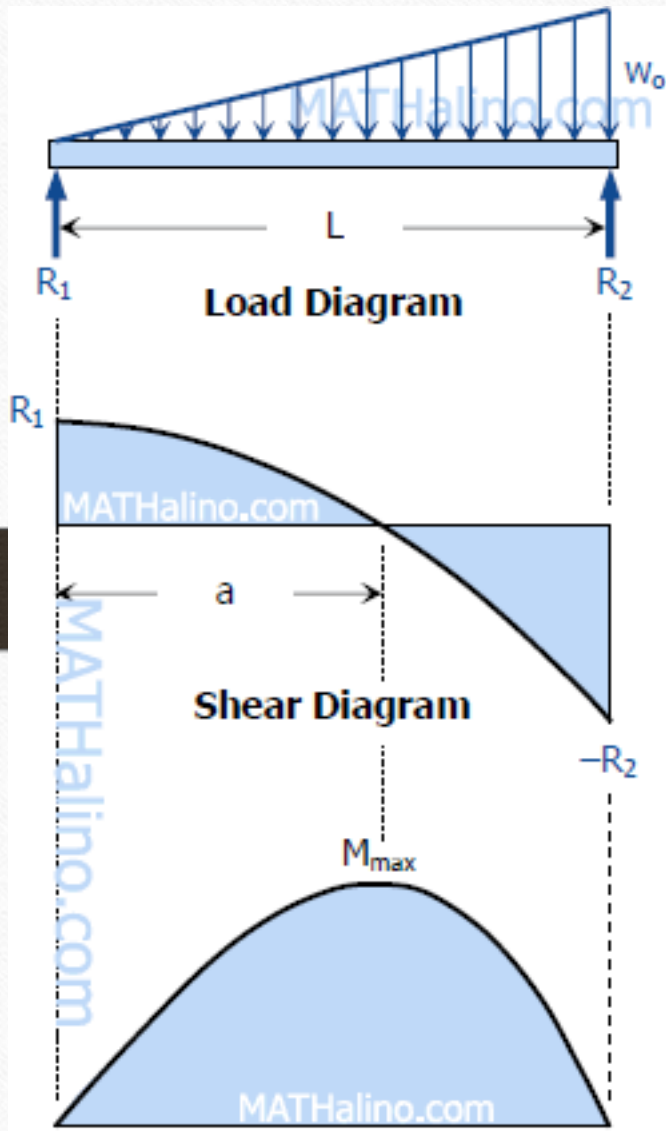


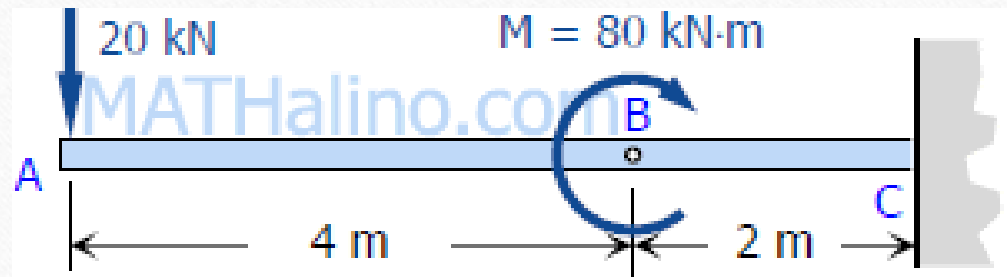




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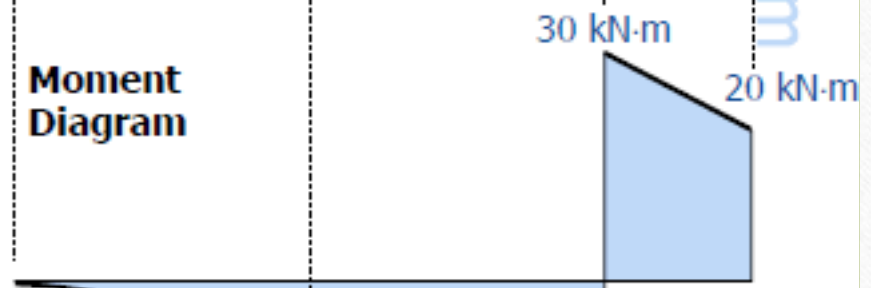
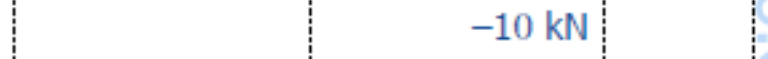
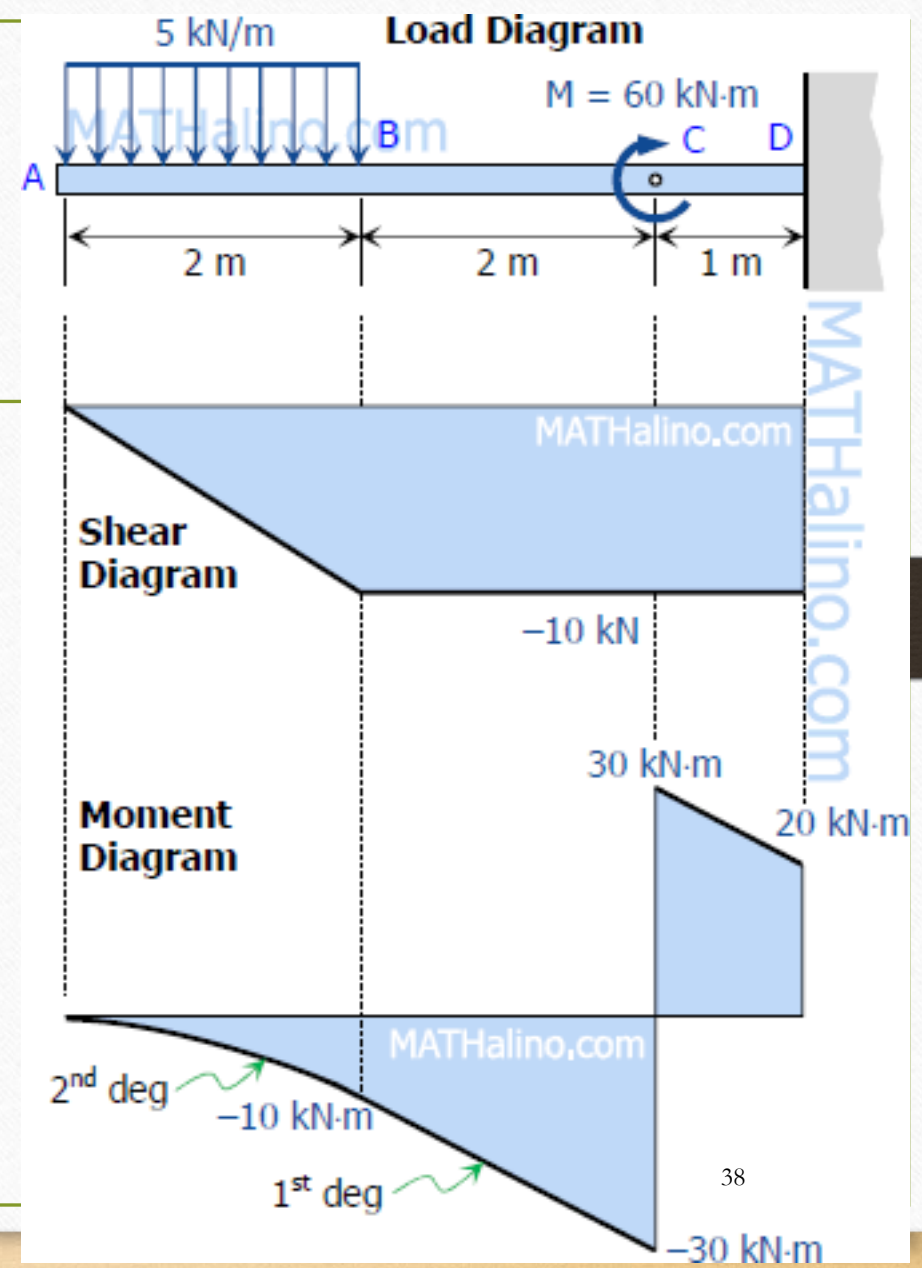


Load Diagram

Shear Diagram

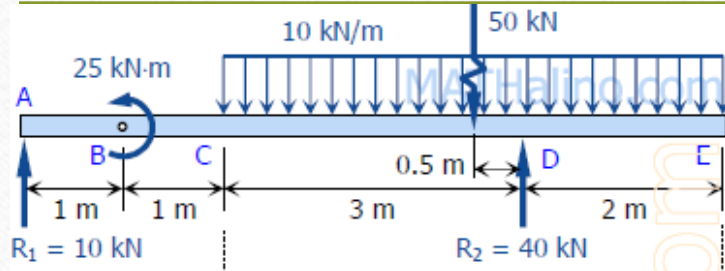


Moment Diagram

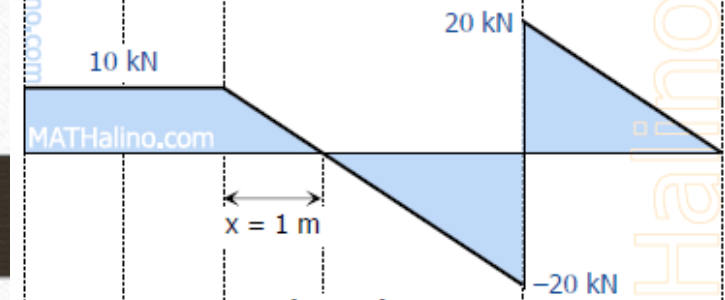


2<sup>nd</sup> deg  
-10 kN-m  
1<sup>st</sup> deg

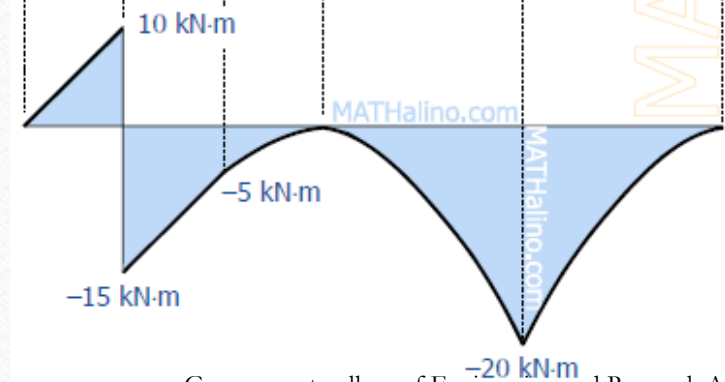




**Load Diagram**

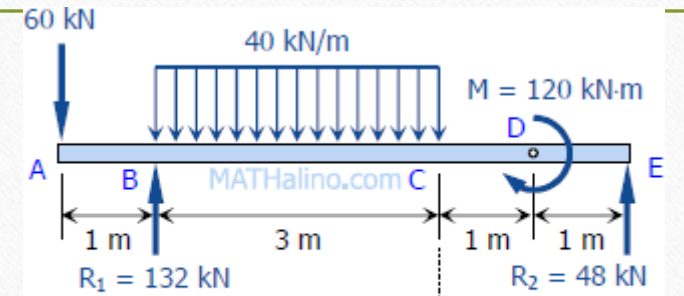


**Shear Diagram**

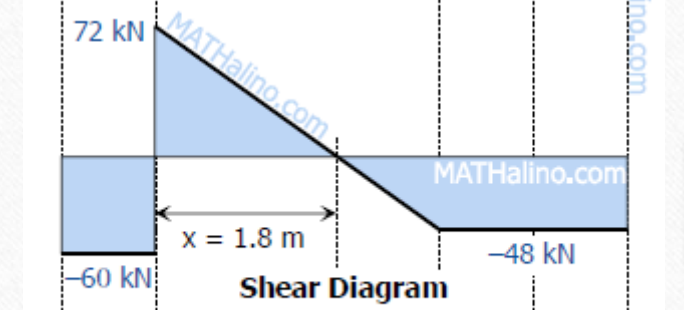


**Moment Diagram**

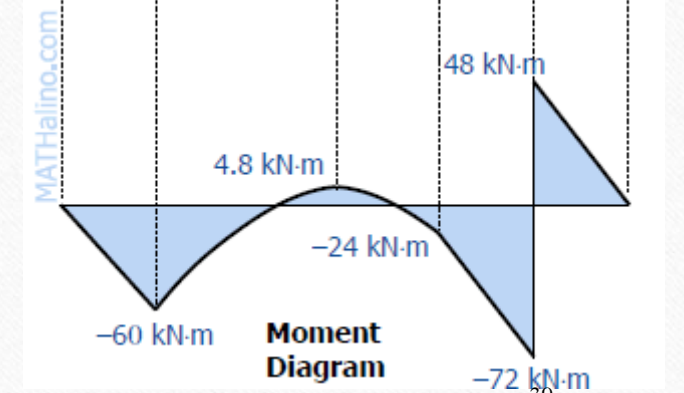
Government college of Engineering and Research Awasari(kh), Pune



**Load Diagram**



**Shear Diagram**



**Moment Diagram**

# Interpretations

Types of Loading	Shape of Shear Force Diagram	Shape of Bending Moment Diagram
Point Load	Sudden Jump at load and constant at other locations	Linearly Varying Graph (Straight Line)
Uniformly Distributed Load (UDL)	Linearly Varying Graph (Straight Line)	Parabolic Graph (Smooth Curve)
Uniformly Varying Load (UDL)	Parabolic Graph (Smooth Curve)	Cubically varying Graph (Curve)
Moment Couple	No effect	Sudden Jump at Moment and constant at other locations

# Relationship between shear force, bending moment

$$\frac{M}{I} = \frac{E}{R} \text{ or } M = \frac{EI}{R}$$

so the basic differentiat

$$M = EI \frac{d^2 y}{dx^2}$$

$$\frac{dM}{dx} = EI \frac{d^3 y}{dx^3} \quad \text{Recalling } \frac{dM}{dx} = F$$

Thus,

$$F = EI \frac{d^3 y}{dx^3}$$

$$\text{i.e } w = - \frac{dF}{dx}$$

$$w = -EI \frac{d^4 y}{dx^4}$$

$$\text{slope} = \frac{dy}{dx}$$

$$\text{B.M} = EI \frac{d^2 y}{dx^2}$$

$$\text{Shear force} = EI \frac{d^3 y}{dx^3}$$

$$\text{load distribution} = EI \frac{d^4 y}{dx^4}$$